

# Re70: Gradients and Partial Derivatives Part 1

## (AIMA4e pp. 119–122)

retraice.com

*The math of ‘Local Search in Continuous Spaces’.*

Air date: Sunday, 4th Dec. 2022, 11:00 PM Eastern/US.

We’re focusing on the math and code difficulties of AIMA4e<sup>1</sup> right now, December 2022. This is in service of our plan to deep-dive the book from Jan.–Jun., 2023.

**DISCLAIMER:** The below notes cannot really be trusted, as they’re a student’s attempt, not an instructor’s.

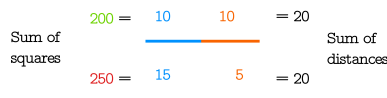
## Placing three airports in Romania

The problem given<sup>2</sup> is to minimize the sum of the squares of the distances (see below comment\*) between each major city in Romania<sup>3</sup> and one of its three to-be-constructed airports. This defines our ‘objective function’  $f$  of  $n$ -dimensional state vector  $\mathbf{x}$ :

$$f(\mathbf{x}) = f(x_1, y_1, x_2, y_2, x_3, y_3)$$

The  $x_i$  and  $y_i$  values are the Cartesian coordinates of the candidate locations of the three airports, from which we can calculate the squares of the distances from each city to its nearest airport and sum them to find the value of our objective function for a given set of airport locations.

**\*Why the squares of the distances?** Perhaps because, as in the case of two cities and one airport, zero-sum arithmetic obtains if we only measure the distances themselves. Taking the squares reflects that some changes in the location of the airport are better than others.



Sums of *squares* reflect ‘better’ airport locations; sums of *distances* don’t necessarily do so.

## Gradient descent

The solution to the airport problem, we’re told, might be found by means of a **gradient vector** and **gradient descent**, i.e. cost minimization. This falls under the math subfield of ‘vector calculus’. The **gradient** is a vector, composed of **partial derivatives** of the **objective function** with respect to each of the six independent variables  $(x_1, y_1, x_2, y_2, x_3, y_3)$ , “that gives the magnitude and direction of the steepest slope” in our state space, i.e. the direction of change at any point in the six-dimensional space of solutions that would locally improve the value of our sum-of-squares objective function.<sup>4</sup>

$$\nabla f = \left( \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$$

Ultimately, gradient descent is about calculating the optimal direction to go in a state space to obtain better states as valued by an objective function, which itself is analogous to ‘elevation’ in the state space. Gradient ‘descent’ and ‘ascent’ are inverses of each other: to minimize the cost, ‘descend’ toward lower elevation; to maximize the benefit, invert the same objective function and ‘descend’ toward higher numbers. Alternatively, ‘ascend’ toward higher cost savings (‘bigger’ negative numbers) or ‘ascend’ toward higher benefits (bigger positive numbers). This works locally, but not necessarily globally, i.e. local maxima and minima are a problem.

## Struggling

We haven’t yet made key decisions about how to tackle difficult topics such as these on Retraice. Depth first? Breadth first? How thorough and explanatory should we be? Our ECMP (English, code, math, progress) method might be a guide, but it’s not the whole answer.<sup>5</sup>

□

<sup>1</sup>Russell & Norvig (2020).

<sup>2</sup>Russell & Norvig (2020) p. 120.

<sup>3</sup>See the map on p. 64, figure 3.1, or p. 89, figure 3.20 (with ‘con-tours’) or at <http://aima.cs.berkeley.edu/figures.pdf>.

<sup>4</sup>Russell & Norvig (2020) p. 120.

<sup>5</sup>Retraice (2022/12/03).

## References

Retraice (2022/12/03). Re69: TABLE-DRIVEN-AGENT Part 5 (ECMP and AIMA4e p. 48). *retraice.com*.  
<https://www.retraice.com/segments/re69> Retrieved 4th Dec. 2022.

Russell, S., & Norvig, P. (2020). *Artificial Intelligence: A Modern Approach*. Pearson, 4th ed. ISBN: 978-0134610993. Searches:  
<https://www.amazon.com/s?k=978-0134610993>  
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