Re76: Gradients and Partial Derivatives Part 7 (AIMA4e pp. 119–122)

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Moving the airport to improve its value.

Air date: Saturday, 10th Dec. 2022, 11:00 PM Eastern/US. We're focusing on the math and code of AIMA4e¹ right now, December 2022. This is in service of our plan to deep-dive the book from Jan.–Jun., 2023. DISCLAIMER: The below mathematics cannot be trusted; it's a student's attempt, not an expert's.

Two more guesses, on the basis of gradient descent:



Lucky-unlucky: There is exactly *one* line (y = x - 7) on which placing an airport leads to the two partial derivatives being equal—and I placed the first airport on it. This lead to tens of minutes of doubt and disappointment, but ultimately I've become reasonably sure I understand the cause and that my calculations are correct.

+ 14+ + 15. 2022 12103 1210 2022 f (STATE X) IX. (STATE = X3) XIV Y. $f(16.5, 9.5) = (16.5 - 14)^2 + (9.5 - 13)^3$ (x -18) 00 TI STI RESPECT TO Y. ²+(18-19)⁴+(1)+k-6)² f (18, 11+h) = (18-14)2 + (16.5-19)"+ (9.5-6) Xa ya + (18-×10)2 + (11- 40)2 + (h - Z) + (-1) + (h + 5) AIRPORT = (4)² $= (2.5)^{2} + (-3.5)^{2}$ + 100 WITH RESPECT TO X. - 16 + 1 + (-2.5)" + (3.5)" h2-4h+4 674 106 <u>9 f</u> (Xa+ A H2+10++25 -9X0 6-26 6.25 + 12.25 h Zh" +6h+29 + 17 + 12.25 + 6.25 WITH RESPECT TO Y. (Xo=18) = 12.5 + 24.5 (49) 9t f (xa y + k) 1m 57) - AND I'S HALF 1-20 dya h THE SWM OF THE SOUTH OF THE DISTANCE BEINERN THE CITIES : + (STATE = X -) IX GETT? ZNO AIRPORT BUESS - (19-H)2 + (13-6)2 14 (EI - 01) + "(HI - 1) 18 f/17,10)= + 49 37 = 25 + (17 - 19)2 + (10 -6)² SLOPES SHEVLY BE ZERD? $= (3)^{2} + (-3)^{2} + (-2)^{2} + (9)^{3}$ 7.0.6 R + 4 + 16 <u>ð</u>f 9 .blad blad "i FOR FROM RETS Plat -E FITER 68) 9% e+ 4+ (-1+1)"+ (4+1) H PESELT TO X. X (17+6,10) - 38 2F = 110 9t 17 + 16+8h+ h JyA -Zh+h* $\frac{\int (|6.5+h,9.5)}{\int (|6.5+h,9.5)} = \frac{(|6.5+h-19|)^2 + (9.5-13)^2}{+(|6.5+h-19|)^2 + (9.5-6)^2}$ - 1in 100 REVERSE FOIL ٢ $\frac{\partial y_{h} \in \mathbb{W}}{\left[\frac{|y_{h}|^{2} + |y_{h}|^{2}}{2} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + (|y_{h}|^{2} + |y_{h}|^{2})^{2}} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2}}{2} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2}} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2}}{2} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2}} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2}}{2} + \frac{|(y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + |y_{h}|^{2} + \frac{|y_{h}|^{2} + |y_{h}|^{2} + |y$ dy. $= (h+2.5)^{2} + (h-2.5)^{2} + (-3.5)^{2} + (3.5)^{2}$ + 64 = h-+5++6.25 +12.25+12.25 + h--5++6.25 = (1+3) = +(1-2) = +9 + 16. $\begin{array}{c} = & h^{2-1} d h h^{2} \\ & + h^{2--1} d h & + 2 \\ \hline & + h^{2--1} d h & + 2 \\ \hline & + h^{2--1} d h & + 2 \\ \hline & + (h^{2-1})^{2--1} (h^{2-1})^{2--1} (h^{2-1})^{2--1} (h^{2-1})^{2--1} \\ \hline & + (h^{2-1})^{2--1} d h \\ \hline & + (h^{2-1})^{2--1} (h^{2-1})^{2--1} (h^{2-1})^{2--1} \\ \hline & + (h^{2-1})^{2--1} (h^{2-1})^{2--1} (h^{2-1})^{2--1} \\ \hline & + (h^{2-1})^{2--1} (h^{2-1})^{2--1} (h^{2-1})^{2--1} \\ \hline & + (h^{2-1})$ = ZL2 + 12.5 + 24.5 = ZL2 + 37 5 6000 = 212 $f(16.5, 9.5+h) = \frac{(165-14)^2}{(165-14)^2} + \frac{(454-15)^2}{(454-65)^2}$ T VIL $= (1 - 3)^{2} + (1 + 1)^{2} + (3)^{2} + (-2)^{2}$ $= (1, -3, 5)^{2} + (1 + 3, 5)^{2} + (2 - 5)^{2} + (-2, 5)^{2}$ = 1 - - 44 + 9 + 9 + 4 = 26 + 26 + 38 - h2-76+12.25 + 6.25 + 6.25 46)(6= 0) + 37 OF IOLINE = 262 SPREADTHEFT SLOPE IS ZERON

The hand-math:

¹Russell & Norvig (2020).

The spreadsheet-math:

∂ of f with respect to x_airport at state x_1		f(18+h, 11) =			f(18+h, 11) - f(18,11) =				
∂f = ∂x_a	lim h -> 0	2h^2	+6h	+46	total	-46	total	/h	
	1	2	6	46	54	-46	8	8	
	0.5	0.5	3	46	49.5	-46	3.5	7	
	0.25	0.125	1.5	46	47.625	-46	1.625	6.5	
	0.125	0.03125	0.75	46	46.78125	-46	0.78125	6.25	
	0.0625	0.0078125	0.375	46	46.3828125	-46	0.3828125	6.125	
	0.03125	0.001953125	0.1875	46	46.18945313	-46	0.189453125	6.0625	
	0.015625	0.00048828125	0.09375	46	46.09423828	-46	0.094238281250	6.03125	
	0.0078125	0.0001220703125	0.046875	46	46.04699707	-46	0.046997070313	6.015625	
	0.00390625	0.00003051757813	0.0234375	46	46.02346802	-46	0.023468017578	6.0078125	
	0.001953125	0.000007629394531	0.01171875	46	46.01172638	-46	0.011726379395	6.00390625	
	0.0009765625	0.000001907348633	0.005859375	46	46.00586128	-46	0.005861282349	6.001953125	
	0.00048828125	0.0000004768371582	0.0029296875	46	46.00293016	-46	0.002930164337	6.000976563	
	0.000244140625	0.0000001192092896	0.00146484375	46	46.00146496	-46	0.001464962959	6.000488281	
	0.000122070312	0.0000002980232239	0.000732421875	46	46.00073245	-46	0.000732451677	6.000244141	
	0.000061035156	0.00000007450580597	0.0003662109375	46	46.00036622	-46	0.000366218388	6.000122070	
	0.000030517578	0.00000001862645149	0.0001831054688	46	46.00018311	-46	0.000183107331	6.000061035	
	0.000015258789	0.000000004656612873	0.00009155273438	46	46.00009155	-46	0.00091553200	6.000030518	
	0.000007629394	0.000000001164153218	0.00004577636719	46	46.00004578	-46	0.000045776484	6.000015259	
								6	∂f =
									ox_a
∂ of f with respect to x_airport at state \bar{x}_3		f(16.5+h, 9.5) =				f(16.5	+h, 9.5) - f(16.5,9.5) =		
∂f = ∂x_a	lim h -> 0	2h^2		37	total	-37	total	/h	
	1	2		37	39	-37	2	2	
	0.5	0.5		37	37.5	-37	0.5	1	
	0.25	0.125		37	37.125	-37	0.125	0.5	
	0.125	0.03125		37	37.03125	-37	0.03125	0.25	
	0.0625	0.0078125		37	37.0078125	-37	0.0078125	0.125	
	0.03125	0.001953125		37	37.00195313	-37	0.001953125	0.0625	
	0.015625	0.00048828125		37	37.00048828	-37	0.00048828125	0.03125	
	0.0078125	0.0001220703125		37	37.00012207	-37	0.0001220703125	0.015625	
	0.00390625	0.00003051757813		37	37.00003052	-37	0.00003051757813	0.0078125	
	0.001953125	0.000007629394531		37	37.00000763	-37	0.000007629394531	0.00390625	
	0.0009765625	0.000001907348633		37	37.00000191	-37	0.000001907348633	0.001953125	
	0.00048828125	0.0000004768371582		37	37.00000048	-37	0.0000004768371582	0.0009765625	
	0.000244140625	0.0000001192092896		37	37.00000012	-37	0.0000001192092896	0.00048828125	
	0.000122070312	0.0000002980232239		37	37.0000003	-37	0.0000002980232239	0.000244140625	
	0.000061035156	0.00000007450580597		37	37.0000001	-37	0.00000007450580597	0.000122070312	
	0.000030517578	0.00000001862645149		37	37	-37	0.00000001862645149	0.000061035156	
	0.000015258789	0.000000004656612873		37	37	-37	0.000000004656612873	0.000030517578	
	0.000007629394	0.000000001164153218		37	37	-37	0.000000001164153218	0.000015258789	
								0	=

The gradient descent tutorial I mentioned: https://realpython.com/gradient-descent-algorithm-python. We might come back and do this.

References

Russell, S., & Norvig, P. (2020). Artificial Intelligence: A Modern Approach. Pearson, 4th ed. ISBN: 978-0134610993. Searches: https://www.amazon.com/s?k=978-0134610993 https://www.google.com/search?q=isbn+978-0134610993 https://lccn.loc.gov/2019047498