Re76: *Gradients and Partial Derivatives Part 7 (AIMA4e pp. 119–122)*

retraice.com

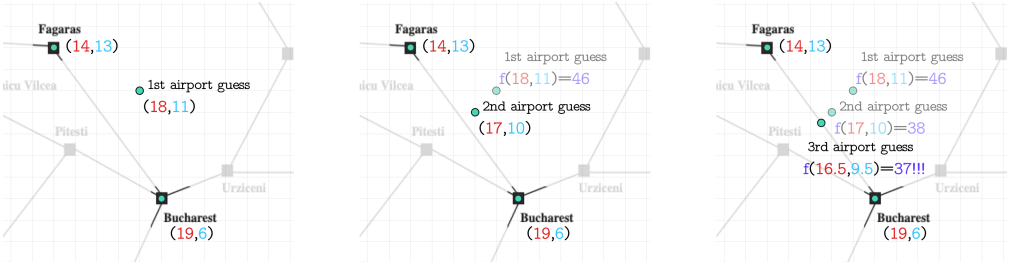
Moving the airport to improve its value.

Air date: Saturday, 10th Dec. 2022, 11:00 PM Eastern/US.

We're focusing on the math and code of AIMA4e[1] *right now, December 2022.*
This is in service of our plan to deep-dive the book from Jan.–Jun., 2023.

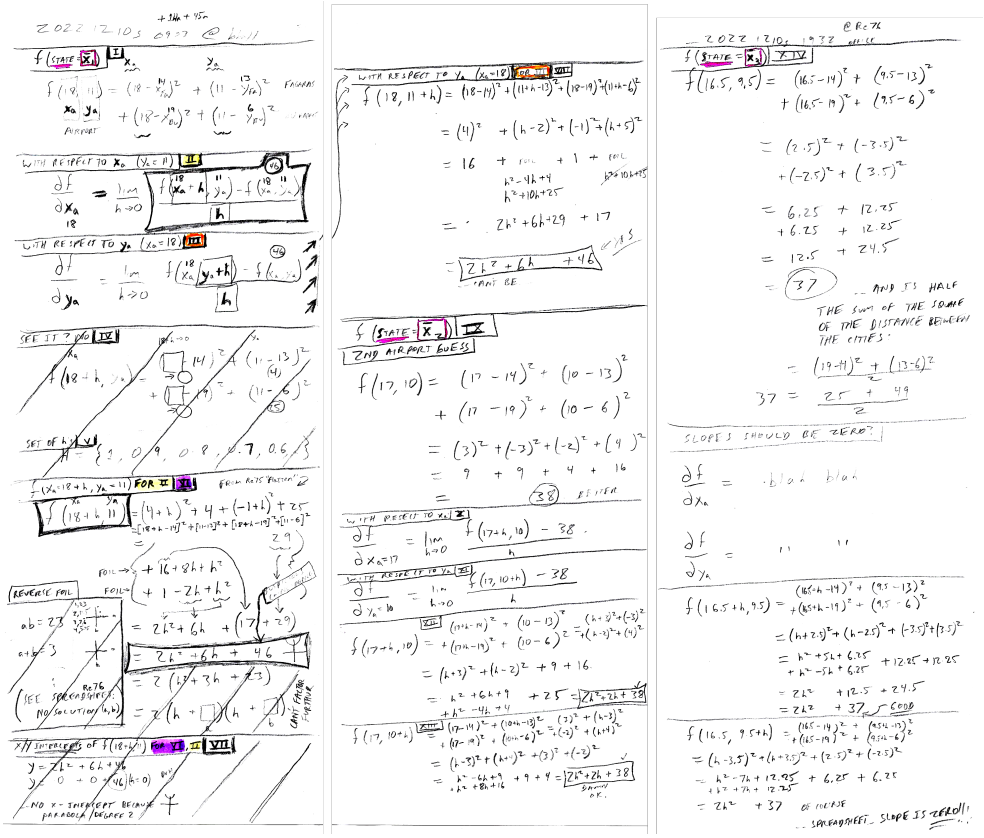
DISCLAIMER: The below mathematics cannot be trusted; it's a student's attempt, not an expert's.

Two more guesses, on the basis of gradient descent:



Lucky-unlucky: There is exactly *one* line ($y = x - 7$) on which placing an airport leads to the two partial derivatives being equal—and I placed the first airport on it. This lead to tens of minutes of doubt and disappointment, but ultimately I've become reasonably sure I understand the cause and that my calculations are correct.

The hand-math:



[1] Russell & Norvig (2020).

The spreadsheet-math:

∂ of f with respect to x_airport at state x̄_1		f(18+h, 11) =				f(18+h, 11) - f(18,11) =			
∂f / ∂x_a =	lim h -> 0	2h^2	+6h	+46	total	-46	total	/h	
	1	2	6	46	54	-46	8	8	
	0.5	0.5	3	46	49.5	-46	3.5	7	
	0.25	0.125	1.5	46	47.625	-46	1.625	6.5	
	0.125	0.03125	0.75	46	46.78125	-46	0.78125	6.25	
	0.0625	0.0078125	0.375	46	46.3828125	-46	0.3828125	6.125	
	0.03125	0.001953125	0.1875	46	46.18945313	-46	0.189453125	6.0625	
	0.015625	0.00048828125	0.09375	46	46.09423828	-46	0.094238281250	6.03125	
	0.0078125	0.0001220703125	0.046875	46	46.04699707	-46	0.046997070313	6.015625	
	0.00390625	0.00003051757813	0.0234375	46	46.02346802	-46	0.023468017578	6.0078125	
	0.001953125	0.000007629394531	0.01171875	46	46.01172638	-46	0.011726379395	6.00390625	
	0.0009765625	0.000001907348633	0.005859375	46	46.00586128	-46	0.005861282349	6.001953125	
	0.00048828125	0.000000476837158	0.0029296875	46	46.00293016	-46	0.002930164337	6.000976563	
	0.000244140625	0.000000119209289	0.00146484375	46	46.00146496	-46	0.001464962959	6.000488281	
	0.000122070312	0.000000029802322	0.000732421875	46	46.00073245	-46	0.000732451677	6.000244141	
	0.000061035156	0.000000007450581	0.000366210938	46	46.00036622	-46	0.000366218388	6.000122070	
	0.000030517578	0.000000001862645	0.000183105469	46	46.00018311	-46	0.000183107331	6.000061035	
	0.000015258789	0.000000000465661	0.000091552734	46	46.00009155	-46	0.000091553200	6.000030518	
	0.000007629394	0.000000000116415	0.000045776367	46	46.00004578	-46	0.000045776484	6.000015259	
								6	∂f = --------- ∂x_a

∂ of f with respect to x_airport at state x̄_3		f(16.5+h, 9.5) =				f(16.5+h, 9.5) - f(16.5,9.5) =			
∂f / ∂x_a =	lim h -> 0	2h^2		37	total	-37	total	/h	
	1	2		37	39	-37	2	2	
	0.5	0.5		37	37.5	-37	0.5	1	
	0.25	0.125		37	37.125	-37	0.125	0.5	
	0.125	0.03125		37	37.03125	-37	0.03125	0.25	
	0.0625	0.0078125		37	37.0078125	-37	0.0078125	0.125	
	0.03125	0.001953125		37	37.00195313	-37	0.001953125	0.0625	
	0.015625	0.00048828125		37	37.00048828	-37	0.00048828125	0.03125	
	0.0078125	0.0001220703125		37	37.00012207	-37	0.0001220703125	0.015625	
	0.00390625	0.00003051757813		37	37.00003052	-37	0.00003051757813	0.0078125	
	0.001953125	0.000007629394531		37	37.00000763	-37	0.000007629394531	0.00390625	
	0.0009765625	0.000001907348633		37	37.00000191	-37	0.000001907348633	0.001953125	
	0.00048828125	0.000000476837158		37	37.00000048	-37	0.000000476837158	0.0009765625	
	0.000244140625	0.000000119209289		37	37.00000012	-37	0.000000119209289	0.00048828125	
	0.000122070312	0.000000029802322		37	37.00000003	-37	0.000000029802322	0.000244140625	
	0.000061035156	0.000000007450581		37	37.00000001	-37	0.000000007450581	0.000122070312	
	0.000030517578	0.000000001862645		37	37	-37	0.000000001862645	0.000061035156	
	0.000015258789	0.000000000465661		37	37	-37	0.000000000465661	0.000030517578	
	0.000007629394	0.000000000116415		37	37	-37	0.000000000116415	0.000015258789	
								0	∂f = --------- ∂x_a

The gradient descent tutorial I mentioned: https://realpython.com/gradient-descent-algorithm-python. We might come back and do this.

□

References

Russell, S., & Norvig, P. (2020). *Artificial Intelligence: A Modern Approach*. Pearson, 4th ed. ISBN: 978-0134610993. Searches:
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